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ORIGINAL RESEARCH

SOFT SET THEORY APPLIED TO MS-ALGEBRAS

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ABSTRACT

In this paper, we initiate the study of soft MS-algebra by using soft set theory. The notions of soft MS-algebras, soft *e*-filters and soft MS-homomorphisms are introduced. Several related properties and some characterizations are investigated.

Keywords/phrases: Soft sets, MS-algebras, Soft MS-algebras, *e*-filters, Soft e-filters.

1. Introduction

Molodtsov introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainty. Because of various uncertainties arise in complicated problems in economics, engineering, environmental science, medical science and social science; methods of classical mathematics may not be successfully used to solve them. Mathematical theories such as probability theory, fuzzy set theory and rough set theory were established by researchers to model uncertainties appearing in the above fields. But all these theories have their own difficulties. To overcome these difficulties, Molodtsov (1999) introduced the concept of soft set as a new Mathematical tool for dealing with uncertainties. As the problem of setting the membership function does not arise in soft set theory, it can be easily applied to many different fields. Some operations on the soft set theory was studied by Maji et al. (2003). Ali *et al.* (2009) studied some new operations on soft sets. Aktas and Cagman (2007) compared soft sets to the related concepts of fuzzy sets and rough sets. They also defined the concept of soft groups and derived some related properties. Nagarajan and Meenambigai (2011) studied soft lattices, soft distributive lattices and soft modular lattices. In this paper, we define the notions of soft MS-algebras, and study some properties of Soft MS-algebras. Moreover, we give several illustrative examples. We define soft MS-algebra homomorphism and obtain some properties. Further, we define soft e-filters of MS-algebras and illustrate them by examples. Throughout the paper, we follow the Molodtsovs' definition soft sets to obtain all the findings.

2 Preliminaries

In this section, we recall some definitions and results which will be used in the paper.

Definition 2.1. (Blyth, 1994; Blyth, 1983) An MS-algebra is an algebra $(L, \vee, \wedge, ^\circ, 0, 1)$ of type $(2, 2, 1, 0, 0)$ such that $(L, \vee, \wedge, 0, 1)$ is a bounded distributive lattice and $a \mapsto a^\circ$ is a unary operation satisfying:

1. $a \leq a^{\circ\circ}$,
2. $(a \wedge b)^\circ = a^\circ \vee b^\circ$,
3. $1^\circ = 0$

Definition 2.2. A Stone algebra is an algebra $\mathbf{S} =< S; \vee, \wedge, ^*, 0, 1 >$ of type $< 2, 2, 1, 0, 0 >$ such that $< S; \vee, \wedge, , 0, 1 >$ is a bounded distributive lattice and $x \mapsto x^*$ is a unary operation on S satisfying the conditions:

1. $x \wedge x^* = 0$;
2. $x \wedge y = 0 \Rightarrow y \leq x^*$;
3. $(x \vee y)^* = x^* \wedge y^*$ and
4. $x^* \vee x^{**} = 1$ for all $x, y \in S$.

Definition 2.3. A de Morgan algebra is an algebra $(L, \vee, \wedge, ^\circ, 0, 1)$ of type $(2, 2, 1, 0, 0)$ such that $(L, \vee, \wedge, 0, 1)$ is a bounded distributive lattice and $a \mapsto \overline{a}$ is a unary operation satisfying:

1. $a^{\circ\circ} = a$;
2. $(a \wedge b)^\circ = a^\circ \vee b^\circ$ and
3. $1^\circ = 0$ for all $a, b \in L$.

Lemma 2.4. *(Blyth, 1983; Blyth, 1999) For any two elements a, b of an MS-algebra, we have the following :*

(1) $0^\circ = 1$

(2) $a \leq b \Rightarrow b^\circ \leq a^\circ$

(3) $a^{\circ\circ\circ} = a^\circ$

(4) $(a \vee b)^\circ = a^\circ \wedge b^\circ$

(5) $(a \vee b)^{\circ\circ} = a^{\circ\circ} \vee b^{\circ\circ}$

(6) $(a \wedge b)^{\circ\circ} = a^{\circ\circ} \wedge b^{\circ\circ}$

Definition 2.5. (Roa, 2014) For any filter F of an MS-algebra M, define F^e as the set $F^e = \{x \in M / x^\circ \leq a^\circ \ for\ some\ a \in F\}$

Definition 2.6. (Roa, 2014) A filter F of an MS-algebra M is called an e-filter of F if $F = F^e$

Definition 2.7. (Molodtsov, 2014) Let U be a nonempty finite set of objects called Universe and let E be a nonempty set of parameters. An ordered pair (F, E) is said to be a Soft set over U, where F is a mapping from E into the set of all subsets of the set U. That is, $F : E \to P(U)$.

Definition 2.8. (Molodtsov, 2014) Let (F, A) and (G, B) be two soft sets over the common universe U. We say that (F, A) is a soft subset of (G, B) if

1. $A \subseteq B$,
2. For all $e \in A$, $F(e) \subseteq G(e)$.

Definition 2.9. (Maji, 2003)
Let (F, A) and (G, B) be two soft sets over the common universe U. The union of two soft sets (F, A) and (G, B) is the soft set (H, C) where $C = A \cup B$ and H is defined as follows:

$$H(e) = \begin{cases} F(e) & for\ e \in A - B, \\ G(e) & for\ e \in B - A, \\ F(e) \cup G(e) & for\ e \in A \cap B, \end{cases}$$

Definition 2.10. (Maji, 2003) Let (F, A) and (G, B) be two soft sets over the common universe U such that $A \cap B \neq \emptyset$. The intersection of (F, A) and (G, B) is the soft set (H, C), where $C = A \cap B$ and $H(e) = F(e) \cap G(e)$, for all $e \in C$.

Definition 2.11. (Maji, 2003) Let (F, A) and (G, B) be two soft sets over the common universe U. Then (F, A) AND (G, B) denoted by $(F, A) \bigwedge (G, B)$ and is defined by $(F, A) \bigwedge (G, B) = (H, A \times B)$ where $H((\alpha, \beta)) = F(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \in A \times B$.

Definition 2.12. (Maji, 2003) Let (F, A) and (G, B) be two soft sets over the common universe U. Then (F, A) OR (G, B) denoted by $(F, A) \bigvee (G, B)$ and is defined by $(F, A) \bigvee (G, B) = (H, A \times B)$, where $H((\alpha, \beta)) = F(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$.

3 Soft MS-algebras

In this section, we give the definition of soft MS-algebras and some properties of soft MS-algebras. Throughout this section, M is an MS-algebras and A is any non empty set. R will refer to an arbitrary binary relation between elements of A and elements of M. That is, $R \subseteq A \times M$. A set-valued function $F : A \to P(M)$ can be defined as $F(x) = \{y \in M : xRy,\ x \in A\}$. The pair (F, A) is a soft set over M. Defining a set-valued function from A to M also defines a binary relation R on $A \times M$, given by $R = \{(x, y) \in A \times M : y \in F(x)\}$.

Definition 3.1. Let (F, A) be a soft set over M. Then (F, A) is said to be a soft MS-algebra over M if $F(x)$ is a subMS-algebra of M, for all $x \in A$.

The set of all soft MS-algebra of M is denoted by $\mathcal{S}_{MS}(M)$. Let us verify this definition using the following examples.

Example 3.2. *Consider the MS-algebra M as shown in Figure 1. Let $A = M$. Define the set-valued function $F : A \to P(M)$ by $F(x) = \{y \in M : xRy \Leftrightarrow x \wedge y^\circ \wedge y^{\circ\circ} = 0,\ x \in M\}$. Then $f(0) = M$, $F(t) = F(x) = F(1) = F(z) = F(u) = F(y) = \{0, 1\}$. Therefore, $F(x)$ is a subMS-algebra of M, for all $x \in A$. Hence $(F, A) \in \mathcal{S}_{MS}(M)$.*

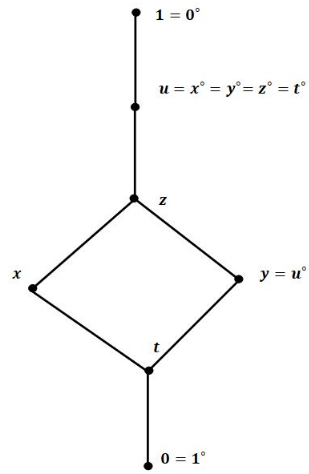

Figure-1

Example 3.3. *Consider the MS-algebra M as shown in Figure-2. Let $A = \{0, a, b, c, d, 1\}$. Define the set-valued function $F : A \to P(M)$ by $F(x) = \{y \in M : xRy \Leftrightarrow x^\circ \vee y^\circ = 1\}$. Then $F(0) = M, F(1) = \{0, 1\}, F(a) = F(b) = F(c) = \{0\}$. $F(a), F(b), F(c)$ are not an MS-algebra, because they do not contain 1. Hence (F, A) is not a soft MS-algebra of M.*

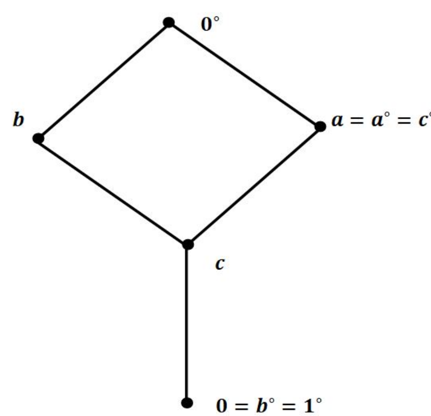

Figure-2

The example given below shows that a soft set which not a soft MS-algebra may contain a soft MS-algebra as soft subset of it.

Example 3.4. *Consider the MS-algebra M in Figure-2. above. Let $B = \{0, 1\}$. Then $F(0) = M, F(1) = \{0, 1\}$. Hence (F, B) is soft MS-algebra of M.*

Remark 3.5. *Every MS-algebra can be considered as a soft MS-algebra.*

Theorem 3.6. *Let $(F, A), (H, B) \in \mathcal{S}_{MS}(M)$ be such that $A \cap B \neq \emptyset$ and $F(z) \cap H(z) \neq \emptyset$ for all $z \in A \cap B$. Then their intersection $(F, A) \cap (H, B) \in \mathcal{S}_{MS}(M)$.*

Proof. The intersection of two soft sets (F, A) and (H, B) is given by $(F, A) \cap (H, B) = (U, C)$ where $C = A \cap B \neq \emptyset$ and $U(z) = F(z) \cap H(z) \neq \emptyset$ for all $z \in C$. Let $x \in C = A \cap B$. Since $x \in A$ and $(F, A) \in \mathcal{S}_{MS}(M)$, $F(x)$ is a subMS-algebra of M. Also since $x \in B$ and $(H, B) \in \mathcal{S}_{MS}(M)$, $H(x)$ is a subMS-algebra of M. By assumption, we have $U(x) = F(x) \cap H(x)$ is a subMS-algebra of M. Since $x \in C$ is arbitrary, $U(x)$ is a subMS-algebra of M, for all $x \in C$. Therefore, $(U, C) = (F, A) \cap (H, B) \in \mathcal{S}_{MS}(M)$. □

Theorem 3.7. *Let $(F, A), (G, B) \in \mathcal{S}_{MS}(M)$. If $A \cap B = \emptyset$, then $(F, A) \cup (G, B) \in \mathcal{S}_{MS}(M)$.*

Proof. The union of two soft sets (F, A) and (G, B) is given by $(F, A) \cup (G, B) = (H, C)$ where $C = A \cup B$ and

$$H(z) = \begin{cases} F(z) & for\ z \in A - B, \\ G(z) & for\ z \in B - A, \\ F(z) \cup G(e) & for\ z \in A \cap B. \end{cases}$$

Since $A \cap B = \emptyset,\ A - B = A,\ B - A = B$, and either $z \in A$ or $z \in B$, for all $z \in A \cup B$.

$$H(z) = \begin{cases} F(z) & for\ z \in A, \\ G(z) & for\ z \in B. \end{cases}$$

Since $(F, A) \in \mathcal{S}_{MS}(M)$, $F(z)$ is a subMS-algebra of M, for all $z \in A$, and also $(G, B) \in \mathcal{S}_{MS}(M)$, $G(z)$ is a subMS-algebra of M, for all $z \in B$. Thus $H(z)$ is a subMS-algebra of M, for all $z \in C$. Therefore, $(H, C) \in \mathcal{S}_{MS}(M)$. That is, $(F, A) \cup (G, B) \in \mathcal{S}_{MS}(M)$. □

Theorem 3.8. *Let $(F, A), (G, B) \in \mathcal{S}_{MS}(M)$ be such that $F(x) \cap G(y) \neq \emptyset$, for all $x \in A$, $y \in B$. Then $(F, A) \bigwedge (G, B) \in \mathcal{S}_{MS}(M)$.*

Proof. The AND operation of two soft sets (F, A) and (G, B) is given by $(F, A) \bigwedge (G, B) = (H, C)$, where $C = A \times B$ and $H((x, y)) = F(x) \cap G(y) \neq \emptyset$, for all $x \in A$, $y \in B$. Since $(F, A) \in \mathcal{S}_{MS}(M)$, $F(x)$ is a subMS-algebra of M, for all $x \in A$. Since $(G, B) \in \mathcal{S}_{MS}(M)$, $G(y)$ is a subMS-algebra of M, for all $y \in B$. Since $F(x) \cap G(y) \neq \emptyset$, it is a subMS-algebra of M, for all $x \in A$, $y \in B$. That is, $H((x, y))$ is a subMS-algebra of M for all $x \in A, y \in B$. Therefore, $(H, C) \in \mathcal{S}_{MS}(M)$. That is, $(F, A) \bigwedge (G, B) \in \mathcal{S}_{MS}(M)$. □

Definition 3.9. Let (F, A) and (H, K) be two soft MS-algebras over$M$. Then (H, K) is a soft subMS-algebra of (F, A) if

1. $K \subseteq A$,
2. $H(x)$ is a subMS-algebra of $F(x)$, for all $x \in K$.

Let us verify this definition using the following examples.

Example 3.10. *Consider the MS-algebra M as shown in Figure-3 below. Let $A = M$, $B = \{0, 1, a, b\}$ and $K = \{0, 1, a\}$. Define a set-valued function $F : A \to P(M)$ by $F(x) = \{y \in M : xRy \Leftrightarrow x \vee y^{\circ} \vee y^{\circ\circ} = 1\}$. Then $F(0) = \{0, 1\}$, $F(1) = M$, $F(a) = \{0, 1, c, d, b\} = F(c), F(b) = \{0, 1, c, a\}, F(d) = \{0, 1, c\}$.*

Define a set-valued function $G : B \to P(M)$ by $G(x) = \{y \in M : xRy \Leftrightarrow x \vee y^{\circ} \vee y^{\circ\circ} = 1, x \in B\}$. Then $G(0) = \{0, 1\}$, $G(1) = M$, $G(a) = \{0, 1, c, d, b\}$ and $G(b) = \{0, 1, c, a\}$. Therefore,

$(F, A), (G, B) \in \mathcal{S}_{MS}(M)$. Here $B \subseteq A$ and $G(x)$ is a subMS-algebra of $F(x)$, for all $x \in B$. Therefore, (G, B) is a soft subMS-algebra of (F, A).

Define a set-valued function $H : K \to P(M)$ by $H(x) = \{y \in M : xRy \Leftrightarrow x \wedge y^\circ \wedge y^{\circ\circ} = 0, x \in K\}$. Then $H(0) = M$, $H(1) = \{0, 1, c\}$, $H(a) = \{0, 1, c, d, b\}$. Therefore, $(F, A), (H, K) \in \mathcal{S}_{MS}(M)$. Here $K \subseteq A$, but $H(0) \nsubseteq G(0)$. Therefore, (H, K) is not a soft subMS-algebra of (F, A).

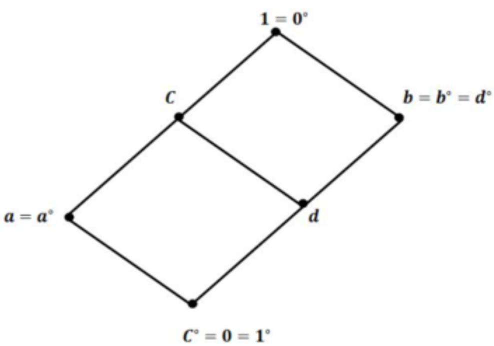

Figure-3

Theorem 3.11. *Let $(F, A), (H, A) \in \mathcal{S}_{MS}(M)$. Then (F, A) is a soft subMS-algebra of (H, A) if and only if $F(x) \subseteq H(x)$, for all $x \in A$.*

Proof. Suppose that (F, A) is a soft subMS-algebra of (H, A). Then $F(x) \subseteq H(x)$, for all $x \in A$. Conversely, let $F(x) \subseteq H(x)$, for all $x \in A$. Since $(F, A) \in \mathcal{S}_{MS}(M)$, $F(x)$ is a subMS-algebra of M, for all $x \in A$. Since $(H, A) \in \mathcal{S}_{MS}(M)$, $H(x)$ is a subMS-algebra of M, for all $x \in A$. Therefore, $F(x)$ becomes a subMS-algebra of $H(x)$, for all $x \in A$. Also $A \subseteq A$. Thus (F, A) is a soft subMS-algebra of (H, A). □

Corollary 3.12. *Every soft MS-algebra is a soft subMS-algebra of itself. That is, if (F, A) is a soft MS-algebra over M, then (F, A) is a soft subMS-algebra of (F, A).*

Definition 3.13. Let f be an MS-algebra homomorphism from L_1 to L_2 and (F, A) be a soft MS-algebra over L_1. Then the image of (F, A) under f is a soft set $(f(F), A)$ defined by:

$$(f(F))(x) = f(F(x)), \text{for all } x \in A.$$

Theorem 3.14. *Let (F, A) and (H, B) be soft MS-algebra over L_1 such that (F, A) be a soft subMS-algebra of (H, B). If f is a homomorphism from L_1 to L_2, then $(f(F), A)$ is a soft subMS-algebra of $(f(H), B)$.*

Proof. Given (F, A) is a soft subMS-algebra of (H, B), then $A \subseteq B$ and $F(x)$ is a subMS-algebra of $H(x)$, for all $x \in A$. Since f is a homomorphism from L_1 to L_2 and homomorphic image of a subMS-algebra in L_1 is a subMS-algebra in L_2, we have $f(F(x))$ and $f(H(y))$ are subMS-algebra of L_2, for all $x \in A$, $y \in B$. Also, $f(F(x))$ is a subMS-algebra of $f(H(x))$, for all $x \in A$. Hence $(f(F), A)$ is a soft subMS-algebra of $(f(H), B)$. □

Let L_1 and L_2 be two MS-algebras with unary $^\circ$. Then $L_1 \times L_2$ is an MS-algebra with respect to the point-wise operations is given by: $(a, b)^\circ = (a^\circ, b^\circ)$.

Definition 3.15. Let (F, A) and (H, B) be two soft MS-algebras over M_1 and M_2, respectively. The product of soft MS-algebras (F, A) and (H, B) is defined as $(F, A) \times (H, B) = (U, A \times B)$, where $U(x, y) = F(x) \times H(y)$ for all $(x, y) \in A \times B$.

Theorem 3.16. *Let (F, A) and (H, B) be two soft MS-algebra over M_1 and M_2, respectively. Then the product $(U, A \times B)$ of (F, A) and (H, B) is a soft MS-algebra over $M_1 \times M_2$.*

Proof. Let $(x, y) \in A \times B$. Then since (F, A) and (H, B) are soft MS-algebra over M_1 and M_2, respectively, we have $F(x)$ is a subMS-algebra over M_1 and $H(y)$ is a subMS-algebra over M_2. Hence it is clear that $F(x) \times H(y)$ is a a subMS-algebra over $M_1 \times M_2$. This proves that the product the product $(U, A \times B)$ of (F, A) and (H, B) is a soft MS-algebra over $M_1 \times M_2$. □

Definition 3.17. Let (F, A) and (H, B) be two soft MS-algebra over M_1 and M_2 respectively. Let $f : M_1 \to M_2$ and $g : A \to B$ be maps. Then (f, g) is said to be a soft lattice homomorphism if

1. f is an MS-algebra homomorphism from M_1 onto M_2,
2. g is a mapping from A onto B,
3. $f(F(x)) = H(g(x))$, for all $x \in A$.

Then (F, A) is said to be a soft MS-algebra homomorphic to (H, B) and it is denoted by $(F, A) \sim (H, B)$. If f is a lattice isomorphism from M_1 onto M_2 and g is a bijection from A to B, then (f, g) is said to be a soft MS-algebra isomorphism. (F, A) is soft MS-algebra isomorphic to (H, B) and it is denoted by $(F, A) \simeq (H, B)$.

Example 3.18. *Consider the MS-algebra L_1 and L_2 as shown in Figure-5 and Figure-6 respectively. Let $A = \{0, a, b, 1\}$ and $B = \{0^{'}, 1^{'}\}$ Define the set-valued function F by $F(x) = \{y \in L_1 : xRy \Leftrightarrow x \wedge y^{\circ} \wedge y^{\circ\circ} = 0, x \in A\}$. Then, $(F, A) \in \mathcal{S}_{MS}(L_1)$. Define the set-valued function H by $H(x) = \{y \in L_2 : xRy \Leftrightarrow x \wedge y^{\circ} \wedge y^{\circ\circ} = 0, x \in B\}$. Then $(H, B) \in \mathcal{S}_{MS}(L_2)$. Define $f : L_1 \to L_2$ by $f(0) = 0^{'}$, $f(a) = 0^{'}$, $f(b) = 1^{'}$, $f(1) = 1^{'}$. Define $g : A \to B$ by $g(0) = 0^{'}$, $g(a) = 0^{'}$, $g(b) = 1^{'}$, $g(1) = 1^{'}$. Then f is MS-algebra homomorphism from L_1 onto L_2 and g is a mapping from A onto B. Also $f(F(x)) = H(g(x))$, for all $x \in A$. Hence (F, A) is a soft MS-algebra homomorphic to (H, B).*

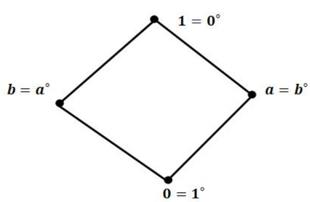

Figure-4

Figure-5

4 Soft *e*-filters of MS-algebras

In this section, we give the definition of soft *e*-filters of MS-algebras and some properties of soft *e*-filters of MS-algebras.

Definition 4.1. Let (F, A) be a soft set over M. Then (F, A) is said to be a soft e-filter over M if $F(x)$ is an e-filters of M, for all $x \in A$.

The set of all soft e-filters of L is denoted by $SF^e(M)$. Let us illustrate this definition using the following examples.

Example 4.2. *Consider an MS-algebras L as shown in Figure 6. Let $A = \{a, b\}$. Define a set-valued function F by $F(a) = \{b \in M : aRb \Leftrightarrow b \wedge a = a\}$. Then $F(a) = \{1, b, a\}, F(b) = \{1, b\}$. Therefore, $F(x)$ is an e-filter of L, for all $x \in A$. Hence $(F, A) \in SF^e(L)$.*

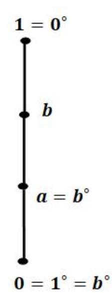

Figure-6

Theorem 4.3. *Let $(F, A), (H, B) \in SF^e(L)$ be such that $A \cap B \neq \emptyset$ and $F(z) \cap H(z) \neq$*

$\emptyset$ for all $z \in A \cap B$. *Then their intersection* $(F, A) \cap (H, B) \in SF^e(L)$.

Proof. The intersection of two soft sets (F, A) and (H, B) is given by $(F, A) \cap (H, B) = (U, C)$ where $C = A \cap B \neq \emptyset$ and $U(z) = F(z) \cap H(z) \neq \emptyset$ for all $z \in C$. Let $x \in C = A \cap B$. Since $x \in A$ and $(F, A) \in SF^e(M)$, $F(x)$ is an e-filter of M. Also since $x \in B$ and $(H, B) \in SF^e(M)$, $H(x)$ is an e-filter of M. By assumption, we have $U(x) = F(x) \cap H(x)$ is an e-filter of M. Since $x \in C$ is arbitrary, $U(x)$ is an e-filter of M, for all $x \in C$. Therefore, $(U, C) = (F, A) \cap (H, B) \in SF^e(M)$. □

Theorem 4.4. *Let* $(F, A), (G, B) \in SF^e(L)$. *If* $A \cap B \neq \emptyset$, *then* $(F, A) \cup (G, B) \in SF^e(L)$.

Proof. The union of two soft sets (F, A) and (G, B) is given by $(F, A) \cup (G, B) = (H, C)$ where $C = A \cup B$ and

$$H(z) = \begin{cases} F(z) & for\ z \in A - B, \\ G(z) & for\ z \in B - A, \\ F(z) \cup G(e) & for\ z \in A \cap B. \end{cases}$$

Since $A \cap B = \emptyset$, $A - B = A$, $B - A = B$, and either $z \in A$ or $z \in B$, for all $z \in A \cup B$.

$$H(z) = \begin{cases} F(z) & for\ z \in A, \\ G(z) & for\ z \in B. \end{cases}$$

Since $(F, A) \in SF^e(M)$, $F(z)$ is an e-filter of M, for all $z \in A$, and also $(G, B) \in SF^e(M)$, $G(z)$ is an e-filter of M, for all $z \in B$. Thus $H(z)$ is an e-filter of M, for all $z \in C$. Therefore, $(H, C) \in SF^e(M)$. That is, $(F, A) \cup (G, B) \in SF^e(M)$. □

Theorem 4.5. *Let* $(F, A), (G, B) \in SF^e(L)$ *be such that* $F(x) \cap G(y) \neq \emptyset$, *for all* $x \in A, y \in B$. *Then* $(F, A) \bigwedge (G, B) \in SF^e(L)$.

Proof. The AND operation of two soft sets (F, A) and (G, B) is given by $(F, A) \bigwedge (G, B) = (H, C)$, where $C = A \times B$ and $H((x, y)) = F(x) \cap G(y) \neq \emptyset$, for all $x \in A$, $y \in B$. Since $(F, A) \in SF^e(L)$, $F(x)$ is an e-filter of M, for all $x \in A$. Since $(G, B) \in SF^e(L)$, $G(y)$ is an e-filter of M, for all $y \in B$. Since $F(x) \cap G(y) \neq \emptyset$, it is an e-filter of M, for all $x \in A$, $y \in B$. That is, $H((x, y))$ is an e-filter of M for all $x \in A, y \in B$. Therefore, $(H, C) \in SF^e(L)$. That is, $(F, A) \bigwedge (G, B) \in SF^e(L)$. □

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