Ethiopian Journal of Natural and computational Science(EJNCS)

Ethiop J Nat Comp Sci 2023, Volume 3, Issue 2

Original Research Article

δ-Ideals in MS-Almost Distributive Lattices

Teferi Getachew Alemayehu[1], Wondwosen Zemene Norahun[2] Gezahagne Mulat Addis[2], Derso Abeje Engidaw[2]

1 Department of Mathematics, Debre Berhan University, Debre Berhan, Ethiopia

2 Department of Mathematics, University of Gondar, Gondar, Ethiopia * Correspondence: wondie1976@gmail.com

Article History:
Received: 09 June 2023
Accepted: 13 August 2023
Published: 20 August 2023

Print ISSN 2710-0200
Electronic ISSN 2710-0219

ABSTRACT

This paper develops the notion of δ-ideals in MS-almost distributive lattices (MS-ADLs), extending the corresponding concept from MS-algebras. It is shown that the family of all δ-ideals forms a complete distributive lattice. In addition, we establish several equivalent criteria for an ideal in an MS-ADL to be a δ-ideal, and investigate the behavior of δ-ideals under homomorphic mappings.

Keywords: MS-algebras, almost distributive lattices, δ-ideals, MS-ADLs, homomorphisms

1 Introduction

Swamy and Rao (1981) introduced the concept of almost distributive lattices (ADLs) as a common framework that unifies ring-theoretic and lattice-theoretic extensions of Boolean algebras. An almost distributive lattice (ADL) is an algebraic structure equipped with two binary operations, $\vee$ and $\wedge$, satisfying most distributive lattice properties with a least element 0, but not necessarily commutativity or the right distributive law. It was shown that enforcing any one of these missing conditions reduces an ADL to a distributive lattice. Extensions of this theory include pseudo-complemented ADLs Swamy et al. (2000)., the broader class of Stone ADLs Swamy et al. (2003), and subsequent investigations on dominator, closure, and δ-ideals in ADLs Rafi et al. (2014), Rafi et al.(2016).

Parallel developments in distributive lattice theory introduced Ockham algebras, defined as bounded distributive lattices with a dual endomorphism, subsuming Boolean, De Morgan, Kleene, and Stone algebras Blyth et al (1994). Within this class, Blyth and Varlet Blyth et al. (1983) formulated MS-algebras as a common generalization of De Morgan and Stone algebras, later shown to form an equational class with fully characterized subvarieties Blyth and Varlet (1983). More recently, Addis extended these ideas by introducing De Morgan ADLs and MS-almost distributive lattices (MS-ADLs) as abstractions combining De Morgan, Stone, and almost Boolean structures Addis, De Morgan (2020). Motivated by these developments, the present work examines δ-ideals in MS-ADLs, establishes their lattice-theoretic properties, provides equivalent characterizations, and analyzes their behavior under homomorphisms.

2 δ-ideals of MS-ADL

The notion of δ-ideals for MS-algebras was first introduced by Addis (2020). In the present section, we extend this idea to MS-almost distributive lattices (MS-ADLs). Although several results appear formally close to those in the MS-algebra setting, the arguments differ substantially due to the absence of certain properties in ADLs—specifically, the commutativity of "$\vee$" and "$\wedge$," as well as the right distributive law of "$\vee$" over "$\wedge$." For the general notions of almost distributive lattices (ADL), as well as their ideals and filters, readers may consult, Swamy and Rao (1981). The basic definitions of MS-almost distributive lattices (MS-ADL) are adopted from Addis (2020), while the concept of MS-algebra follows Blyth et al. (1994). Unless specified otherwise, throughout this section we denote L as an MS-ADL.

Definition 2.1. If H is a filter, we define a set

$$\delta(H) = \{s \in L : s^\circ \in H\}$$

.

Lemma 2.2. *If H is a filter of L. Then $\delta(H)$ is an ideal of L.*

Proof. Clearly $0 \in \delta(H)$ and thus $\delta(H) \neq \emptyset$. For $s, p \in \delta(H)$, we have $s^\circ, p^\circ \in H$, which yields $s^\circ \wedge p^\circ = (s \vee p)^\circ \in H$. Consequently, $s \vee p \in \delta(H)$. Now, let $s \in \delta(H)$ and $q \in L$.

Then $s^\circ \in H$, and therefore $(s \wedge q)^\circ = (q \wedge s)^\circ = q^\circ \vee s^\circ \in H$, which implies $s \wedge q \in \delta(H)$. Hence, $\delta(H)$ forms an ideal. □

Lemma 2.3. *Let E and H be filters of L. Then*

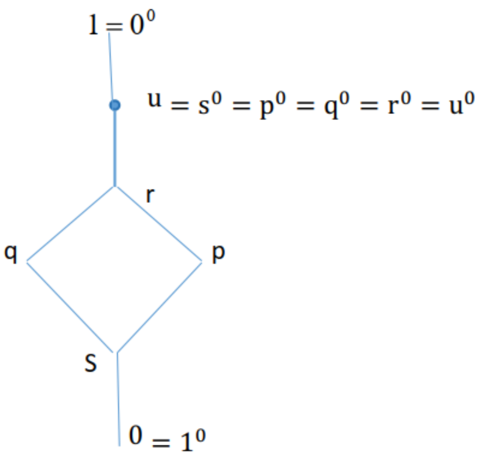
1. $E \cap \delta(E) = \emptyset$, *whenever L is a stone ADL and E is a proper filter,*
2. $s \in \delta(E) \Rightarrow s^{\circ\circ} \in \delta(E)$,
3. $s \in E \Rightarrow s^\circ \in \delta(E)$,
4. $E = L \Leftrightarrow \delta(E) = L$,
5. $E \subseteq H \Rightarrow \delta(E) \subseteq \delta(H)$,
6. $\delta(D) = \{0\}$,
7. *if E is prime, then $\delta(E)$ is a prime,*
8. $\delta(E \cap H) = \delta(E) \cap \delta(H)$.

Proof. (1) If $E \cap \delta(E) \neq \emptyset$, there is $s \in E \cap \delta(E)$, in which $s \in E$ and $s^\circ \in E$. Since E is a filter and L is a Stone ADL, it follows that $s \wedge s^\circ = 0 \in E$, a contradiction. Hence, $E \cap \delta(E) = \emptyset$.
(2) Let $s \in \delta(E)$, then $s^{\circ\circ\circ} = s^\circ \in E$. This implies $s^{\circ\circ} \in \delta(E)$.
(3) Let $s \in E$, then $s = s^{\circ\circ} \wedge s \in E$. This implies $s^{\circ\circ} \in E$. Hence $s^\circ \in \delta(E)$.
(4) Suppose that $E = L$. Then $0 = 0^{\circ\circ} \in E$. This implies $m = 0^\circ \in \delta(E)$. This implies $\delta(E) = L$. Conversely, suppose that $\delta(E) = L$. This implies $m \in \delta(E)$ for any maximal element m of L. Thus $m^\circ = 0 \in E$ and hence $E = L$.
(5) Let $E \subseteq H$ and take $s \in \delta(E)$. Then $s^\circ \in E$, and since $E \subseteq H$, it follows that $s^\circ \in H$. Consequently, $s \in \delta(H)$, and therefore $\delta(E) \subseteq \delta(H)$.
(6) If $s \in \delta(D)$, then $s^\circ \in D$. This implies $s^{\circ\circ} = 0$. This implies $s = s^{\circ\circ} \wedge s = 0 \wedge s = 0$. Hence $\delta(D) = \{0\}$.
(7) Assume E is prime. If $s \wedge p \in \delta(E)$, then $(s \wedge p)^\circ = s^\circ \vee p^\circ \in E$. Since E is prime, it follows that either $s^\circ \in E$ or $p^\circ \in E$, which means $s \in \delta(E)$ or $p \in \delta(E)$. Therefore, $\delta(E)$ is a prime ideal. □

Now we give the definition of δ-ideal in an ADL L.

Definition 2.4. An ideal I of L is called a δ-Ideal if $I = \delta(F)$ for some filter F of L.

Example 2.5. *Let us take a discrete MS-ADL* $A = \{0', v\}$ *together with an MS-algebra* $B = \{0, s, p, q, r, u, 1\}$ *whose Hasse diagram is given below.*



Take $L = A \times B = \{(0', 0), (0', s), (0', p), (0', q), (0', r), (0', u), (0', 1),$ $(v, 0), (v, s), (v, p), (v, q), (v, r), (v, u), (v, 1)\}$. *Then* $(L, \vee, \wedge, ^\circ, \overline{0})$ *is an MS-ADL with zero* $\overline{0} = (0', 0)$ *under point-wise operations. Consider* $I = \{(0', 0), (0', s), (0', p), (0', q), (0', r), (v, 0), (v, s), (v, p)$ *and* $F = \{(0', u), (0', 1), (v, u), (v, 1)\}$. *Clearly, I is an ideal of L and F is a filter of L. Now* $\delta(F) = \{x \in L : x^\circ \in F\} = \{(0', 0), (0', s), (0', p), (0', q),$ $(0', r), (v, 0), (v, s), (v, p), (v, q), (v, r)\} = I$. *Therefore, I is a δ-ideal of L.*

Lemma 2.6. *A proper δ-ideal of L contains no dense element.*

Proof. Let G be a proper δ-ideal of L. By Lemma 2.3(4), there exists a proper filter H of L such that $G = \delta(H)$. We now prove that G contains no dense element. Suppose, on the contrary, that some dense element $s \in \delta(H)$. Then $s^\circ = 0 \in H$, which forces $H = L$, a contradiction. Thus, the claim follows. □

Let $G^\delta(L)$ represent the collection of all δ-ideals of L. The next theorem establishes that $G^\delta(L)$ forms a complete distributive lattice.

Theorem 2.7. *The set $G^\delta(L)$ forms a complete distributive lattice.*

Proof. If $E, H \in L$, we define $\cap$ and $\sqcup$ as:

$$\delta(E) \cap \delta(H) = \delta(E \cap H) \text{ and } \delta(E) \sqcup \delta(H) = \delta(E \vee H).$$

We now show that $\delta(E) \sqcup \delta(H)$ serves as the supremum of $\delta(E)$ and $\delta(H)$ in $G^{\delta}(L)$. Since $E \subseteq E \vee H$ and $H \subseteq E \vee H$, Lemma 2.3(5) ensures that $\delta(E), \delta(H) \subseteq \delta(E \vee H)$. Hence, $\delta(E \vee H)$ is an upper bound of both $\delta(E)$ and $\delta(H)$. Suppose K is a δ-ideal containing $\delta(E)$ and $\delta(H)$. Then $K = \delta(M)$ for some filter M of L, with $\delta(E) \subseteq \delta(M)$ and $\delta(H) \subseteq \delta(M)$. To prove $\delta(E \vee H) \subseteq \delta(M)$, let $s \in \delta(E \vee H)$. Then $s^{\circ} \in E \vee H$, so $s^{\circ} = p \wedge q$ for some $p \in E$ and $q \in H$. By Lemma 2.3(3), it follows that $p^{\circ} \in \delta(E) \subseteq \delta(M)$ and $q^{\circ} \in \delta(H) \subseteq \delta(M)$.

$$\begin{aligned}
&\Rightarrow \quad p^{\circ} \vee q^{\circ} \in \delta(M) \\
&\Rightarrow \quad s^{\circ\circ} = (p \wedge q)^{\circ} \in \delta(M) \\
&\Rightarrow \quad s^{\circ} \in M \\
&\Rightarrow \quad s \in \delta(M)
\end{aligned}$$

Thus $\delta(E \vee H) \subseteq \delta(M)$. So $\delta(E \vee H)$ is the supremum of $\delta(E)$ and $\delta(H)$ in $G^{\delta}(L)$. Hence $(G^{\delta}(L), \cap, \sqcup)$ is a lattice.

To show distributivity, let $\delta(E), \delta(H), \delta(M)$. Then

$$\begin{aligned}
\delta(E) \sqcup (\delta(H) \cap \delta(M)) &= \delta((E) \sqcup (H \cap M)) \\
&= \delta(E \vee H) \cap \delta(E \vee M) \\
&= (E \sqcup H) \cap (E \sqcup M)
\end{aligned}$$

Thus $G^{\delta}(L)$ is distributive.

We now establish completeness. Clearly, 0 and L are the least and greatest elements of $G^{\delta}(L)$. For a subfamily $\delta(E_i) : i \in G \subseteq G^{\delta}(L)$, the intersection $\bigcap_{i \in G} \delta(E_i)$ forms an ideal of L and we can easily show that it is a δ-ideal. Thus $\bigcap_{i \in G} \delta(E_i) \in G^{\delta}(L)$. So $(G^{\delta}(L), \sqcup, \cap)$ is a complete distributive lattice. □

Definition 2.8. A δ-ideal G of L is said to be principal if there exists an element $s \in L$ for which $G = \delta([s))$.

Theorem 2.9. *For any $s \in L$, $(s^{\circ}]$ is a δ-ideal of L.*

Proof. Let $p \in (s^\circ]$. Then $p = s^\circ \wedge p$. This implies $p^\circ = s^{\circ\circ} \vee p^\circ$. This implies $p^\circ \wedge s = (s^{\circ\circ} \vee p^\circ) \wedge s = (s^{\circ\circ} \wedge s) \vee (p^\circ \wedge s) = s \vee (p^\circ \wedge s) = s$. Thus $p^\circ \in [s)$. So $p \in \delta([s))$. Hence $(s^\circ] \subseteq \delta([s))$.
Conversely, let $p \in \delta([s))$. Then $p^\circ \in [s)$ and so $p^\circ \vee s = p^\circ$.

$$\begin{aligned}
&\Rightarrow \quad p^{\circ\circ} \wedge s^\circ = p^{\circ\circ} \\
&\Rightarrow \quad p^{\circ\circ} \wedge s^\circ \wedge p = p^{\circ\circ} \wedge p \\
&\Rightarrow \quad s^\circ \wedge p^{\circ\circ} \wedge p = p^{\circ\circ} \wedge p \\
&\Rightarrow \quad s^\circ \wedge p = p \\
&\Rightarrow \quad p \in (s^\circ]
\end{aligned}$$

Hence $\delta([s)) \subseteq (s^\circ]$. Therefore $\delta([s)) = (s^\circ]$. □

The following results describe certain properties of principal δ-ideals.

Theorem 2.10. *1. $\delta([s)) = (s^\circ]$ for each $s \in L$.*

2. $\delta([s)) = \delta([s^{\circ\circ}))$ for each $s \in L$.

3. $\delta([p)) = \{0\}$ for each $p \in D$.

4. If E is a filter, then $\delta([q)) \subseteq \delta([E)$ for each $q \in E$.

Proof. (1) It is clear.
(2) Since $s^\circ = s^{\circ\circ\circ}$ and by (1), $\delta([s)) = (s^\circ] = (s^{\circ\circ\circ}] = \delta([s^{\circ\circ}))$.
(3) For every $p \in D$, we have $\delta([p)) = (p^\circ] = (0] = \{0\}$
(4) Let $q \in E$. Suppose that $u \in \delta([q))$. Then $u^\circ \in [q)$.

$$\begin{aligned}
&\Rightarrow \quad u^\circ = u^\circ \vee q \in E \\
&\Rightarrow \quad u \in \delta(E)
\end{aligned}$$

Hence for all $q \in E$, $\delta([q)) \subseteq \delta([E)$ for any filter E of L. □

Denote the collection of all principal δ-ideals of an MS-ADL L by $N^\circ(L) = \{\delta([s)) : s \in L\} = \{(s^\circ] : s \in L\}$. The next theorem shows that $M^\circ(L)$ forms a De Morgan algebra.

https://doi.org/10.20372/ejncs/Ma.2023.26

Theorem 2.11. *The set $N^{\circ}(L)$ is a sublattice of $G^{\delta}(L)$ and forms a de Morgan algebra. Moreover, the mapping $s \rightarrow (s^{\circ}]$ is a dual homomorphism of L into $N^{\circ}(L)$.*

Proof. Let $\delta([s)), \delta([p)) \in N^{\circ}(L)$. Then

$$\delta([s)) \cap \delta([p)) = \delta([s \vee p)) \in N^{\circ}(L) \text{ and } \delta([s)) \vee \delta([p)) = \delta([s \vee p)) \in N^{\circ}(L).$$

Moreover, $0 = \delta([n)) \in N^{\circ}(L)$ and $L = \delta([0)) \in N^{\circ}(L)$. Hence, $N^{\circ}(L)$ is a bounded sublattice of $G^{\delta}(L)$ and therefore distributive. Define a unary operation $^{-}$ on $N^{\circ}(L)$ by $\overline{\delta([s))} = \delta([s^{\circ}))$. It is straightforward to check that this operation satisfies the De Morgan laws, showing that $N^{\circ}(L)$ is a De Morgan algebra. Furthermore, the map $s \mapsto (s^{\circ}]$ is a dual homomorphism from L into $N^{\circ}(L)$. □

Theorem 2.12. *For any ideal G of L, the following conditions are equivalent:*

1. *G is a δ-Ideal,*
2. *$G = \cup_{u \in G} \delta([u^{\circ}))$,*
3. *for any s, p in L, $\delta([s^{\circ})) = \delta([p^{\circ}))$ and $s \in I$ imply $p \in I$.*

Proof. (1) ⇒ (2): Suppose G is a δ-ideal. Then $G = \delta(E)$ for some filter E of L. If $s \in G$, we have $s \in \delta(E)$.

$$\begin{aligned} &\Rightarrow \quad s^{\circ\circ\circ} = s^{\circ} \in E \\ &\Rightarrow \quad s^{\circ\circ} \in \delta([s^{\circ})) \\ &\Rightarrow \quad s \in \delta([s^{\circ})) \subseteq \cup_{u \in G} \delta([u^{\circ})) \end{aligned}$$

This implies $G \subseteq \cup_{u \in G} \delta([u^{\circ}))$. Clearly $\cup_{u \in G} \delta([u^{\circ})) \subseteq G$. Hence $G = \cup_{u \in G} \delta([u^{\circ}))$.
(2) ⇒ (3): Suppose condition (2) holds. Let for any s, p in L, $\delta([s^{\circ})) = \delta([p^{\circ}))$ and $s \in G$. Then $\delta([s^{\circ})) = \delta([p^{\circ})) \subseteq G$.

$$\begin{aligned} &\Rightarrow \quad p^{\circ\circ} \in G \\ &\Rightarrow \quad p = p^{\circ\circ} \wedge p \in G \end{aligned}$$

(3) ⇒ (1): Suppose condition (3) holds. Let G be an ideal, define

$$E = \{s \in L : s^{\circ} \in G\}.$$

Take $s, p \in E$. Then $s^\circ, p^\circ \in G$, which gives $(s \wedge p)^\circ \in G$, and hence $s \wedge p \in E$. Next, if $s \in E$ and $q \in L$, we have $s^\circ \in G$ and $s^\circ \wedge q^\circ \in G$, implying $q \vee s \in E$. Hence, E is filter. Moreover, $(q \vee s)^\circ = (s \vee q)^\circ = s^\circ \wedge q^\circ \in G$, confirming again that $q \vee s \in E$ and that E is indeed a filter.

To show $G = \delta(E)$, let $s \in G$. Then $s \in G$ and $\delta([s^\circ)) = \delta([s^{\circ\circ\circ}))$. By condition (3), it follows that $s^{\circ\circ} \in G$.

$$\Rightarrow \quad s^\circ \in E$$
$$\Rightarrow \quad s \in \delta(E)$$

This implies $G \subseteq \delta(E)$.

Conversely, let $s \in \delta(E)$. Then $s^\circ \in E$.

$$\Rightarrow \quad s^{\circ\circ} \in G$$
$$\Rightarrow \quad s = s^{\circ\circ} \wedge s \in G$$

Thus $\delta(E) \subseteq G$. So $\delta(E) = G$. □

3 δ-Ideals and Homomorphisms

In this section, we examine certain characteristics of homomorphic and inverse images of δ-ideals within MS-ADLs. Throughout this section L and M denote MS-ADLs and $f : L \longrightarrow M$ denotes a lattice homomorphism satisfying $(f(s))^\circ = f(s^\circ)$ for every $s \in L$.

Theorem 3.1. *If f is onto, then the following holds:*

(1) *If G is δ-ideal of L, then $f(G)$ is a δ-ideal,*

(2) $u \in L \Rightarrow f(\delta([u))) = \delta(f([u))$*,*

(3) *If G is δ-ideal of L, then $f(G) = \cup_{q \in G} \delta([((f(q))^\circ)))$,*

(4) *If E is a filter of L, then $f(\delta(E)) = \delta(f(E))$.*

Proof. Let G be a δ-ideal of L. Then $G = \delta(E)$ for some filter E of L. Now, it is enough to show that $f(G) = \delta(f(E))$ for some filters of E.

$$\begin{aligned} p \in f(G) = f(\delta(E)) &\Rightarrow p = f(s) \ \ for\ some\ s \in \delta(E) \\ &\Rightarrow s^{\circ} \in E \\ &\Rightarrow p^{\circ} = f(s)^{\circ} \in f(E) \\ &\Rightarrow p \in \delta(f(E)) \end{aligned}$$

This implies $f(G) \subseteq \delta(f(E))$.

Conversely, let $p \in \delta(f(E))$. Then $p^{\circ} = f(s)$ for some $s \in E$. This implies that $s^{\circ\circ} \in E$ and $s^{\circ} \in \delta(E)$. Thus $p^{\circ\circ} = f(s^{\circ}) \in f(\delta(E))$. Since $f(\delta(E))$ is an ideal of N and $p \in N$, we get that $p = p^{\circ\circ} \wedge p \in f(\delta(E))$. Thus $\delta(f(E)) \subseteq f(G)$. So $f(G) = \delta(f(E))$. □

Theorem 3.2. *If f is onto, then the following holds:*

(1) *If G is δ-ideal of M, $f^{-1}(G)$ is a δ-ideal of L,*

(2) *$Ker f$ is a δ-ideal of L.*

Proof. (1) Since G is a δ-ideal of M, there exists a filter E of M such that $G = \delta(E)$. We aim to show that $f^{-1}(G) = \delta(f^{-1}(E))$, where $f^{-1}(G)$ is an ideal of L. Now,

$$\begin{aligned} s \in f^{-1}(G) &\Rightarrow f(s) \in G = \delta(E) \\ &\Rightarrow (f(s))^{\circ} = f(s^{\circ}) \in E \\ &\Rightarrow s^{\circ} \in f^{-1}(E) \\ &\Rightarrow s \in \delta(f^{-1}(E)) \end{aligned}$$

This implies $f^{-1}(G) \subseteq \delta(f^{-1}(E))$.

Conversely,

$$\begin{aligned} s \in \delta(f^{-1}(E)) &\Rightarrow s^{\circ} \in f^{-1}(E) \\ &\Rightarrow (f(s))^{\circ} \in E \\ &\Rightarrow f(s) \in \delta(E) = G \\ &\Rightarrow s \in f^{-1}(G) \\ &\Rightarrow \delta(f^{-1}(E)) \subseteq f^{-1}(G) \end{aligned}$$

Hence $f^{-1}(G)$ is δ-ideal.

(2) Since f is a homomorphism, then $Kerf = \{s \in L : f(s) = 0\}$ is an ideal of L and $Cokerf = \{s \in L : f(s) = m\}$ is a filter of L, for a maximal element m. Easily we prove that $Kerf = \delta(Cokerf)$ and so $Kerf$ is a δ-ideal of L. □

Lemma 3.3. *If f is onto, then the following holds:*

(1) *$G^{\circ}(L)$ is homomorphic of $G^{\circ}(M)$,*

(2) *$G^{\delta}(L)$ is homomorphic of $G^{\delta}(M)$.*

References

[1] G. M. Addis, De Morgan Almost Distributive Lattices, *Southeast Asian Bulletin of Mathematics (2020) 44, 451-464.*

[2] G. M. Addis, MS-almost distributive lattices, *Asian-European Journal of Mathematics, Vol. 13, No. 1 (2020) 2050135 (14 pages)*.

[3] E. Badawy, δ-ideals in MS-algebras.*J compu sci Syst Bio ,9(2) ,2016,28-32*.

[4] J. Berman, Distributive lattices with an additional unary operation. *Aequationes Math. 16, (1977),165-171*.

[5] T. S. Blyth and J.C.Varlet, Ockham Algebras. *Oxford University Press, (1994)*.

[6] T. S. Blyth and J.C. Varlet, On a common abstraction of de Morgan and Stone algebras. *Proc Roy Soc Edinburgh,Sect A, 1983, 94A: 301-308*.

[7] T.S. Blyth and J.C.Varlet, Subvarieties of the class of MS-algebras. *Proc Roy Soc Edinburgh, Sect A, 1983,95A: 157-169*.

[8] N. Rafi, Ravi Kumar Bandaru and G.C. Rao, Dominator and Closure ideals in Almost Distributive Lattices, *Asian-European Journal of Mathematics · June 2014*.

[9] N. Rafi, Ravi Kumar Bandaru and S.N. Rao, δ-ideals in Pseudo-complemented Almost Distributive Lattices, *Palestine Journal of Mathematics, Vol. 5(2) (2016) , 240–248*.

[10] U.M. Swamy and G.C. Rao, Almost Distributive Lattices, *J. Aust. Math. Soc. (Series A), 31 (1981), 77-91*.

[11] U. M. Swamy, G. C. Rao and G. N. Rao, Pseudo-complementation on almost distributive lattices, *Southeast Asian Bull. Math. 24 (2000) 95–104*.

[12] U. M. Swamy, G. C. Rao and G. N. Rao, Stone almost distributive lattices, *Southeast Asian Bull. Math. 27 (2003) 513–526*.